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# One loop effective potential in heterotic $M$-theory

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Abstract: We have calculated the one loop effective potential of the vector multiplets arising from the compactification to five dimensions of heterotic $M$-theory on a Calabi-Yau manifold with $h^{1,1} > 1$. We find that extensive cancellations between the fermionic and bosonic sectors of the theory cause the effective potential to vanish, with the exception of a higher order curvature term of the type which might arise from string corrections.

Keywords: Field Theories in Higher Dimensions, M-Theory.

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## Contents



## 1. Introduction

The weakly coupled $E_8 \times E_8$ heterotic string is one of the most phenomenologically viable of the five consistent string theories. Unfortunately, the predicted value for Newton's constant in this theory is too large. Witten [1] has shown that this situation can be resolved in the *strong* coupling limit, which is believed to be eleven-dimensional supergravity on the orbifold $\mathcal{M}^{10} \times S^1/Z_2$, with $E_8$ Super Yang-Mills gauge theories on the orbifold fixed points [2]. This theory can be compactified on a Calabi-Yau three-fold to obtain six internal dimensions. It is known that in order for the theory to predict the correct known values of Newton's constant and grand unification gauge couplings, the orbifold radius must be an order of magnitude or so larger than the Calabi-Yau compactification scale. Hence, at some intermediate energy scale, the theory has a consistent five-dimensional description.

Lukas *et al.* [3, 4] have derived the five-dimensional effective action by reducing Hořava-Witten theory on a Calabi-Yau space. They have shown that the resulting theory is a gauged version of $N = 1$ supergravity in five dimensions, with a non-abelian set of $E_8$ gauge fields on one orbifold plane, spontaneously broken to $E_6$, and an unbroken $E_8$ gauge group on the other. The vacuum solution for this theory has a domain wall structure with a curved bulk metric. This was the predecessor of the "brane-world" scenarios [5].

In addition to a gravitational and universal hypermultiplet present for every Calabi-Yau compactification, the five-dimensional theory contains a number of vector multiplets, depending on the topological properties of the Calabi-Yau. More precisely, there are $n_V = h^{1,1} - 1$ of these vector multiplets. The background solution is a multi-charged BPS domain wall with $h^{1,1}$ charges. Lukas *et al.* show that there always exists a background solution with $h^{1,1} \geq 1$ with two moduli parameters representing the separation of the domain walls and the dilaton field.

In this paper, we will calculate the one loop effective potential of these vector multiplets about this simple background solution as a function of the moduli.

The vector multiplets decouple from the gravitational and universal hypermultiplets making them the simplest multiplets for the one loop calculation. Furthermore, it is possible to change the number of vector hypermultiplets by changing $h^{1,1}$ without altering the background solution or any of the other multiplets at one loop order. Therefore, the vector multiplets tell us how the total vacuum energy depends on the topology of the Calabi-Yau manifold.

The five dimensional domain wall solution gives rise to an effective four dimensional theory in which the separation of the domain walls becomes one of the moduli fields. It is important to identify effects which can provide a potential for the brane separation and fix this particular modulus. One possible mechanism is that quantum fluctuations of the bulk fields stabilise the branes at phenomenologically acceptable positions. This has been discussed extensively in the context of the Randall-Sundrum brane world scenario [6]–[10]. Previous work of this kind in five-dimensional heterotic $M$-theory has been done for scalar fields by Garriga et al. [11].

The BPS solution described above preserves half of the supersymmetry, hence one would expect cancellation of the contributions to the effective action of the bosonic and fermionic sectors. In a related calculation in an eleven dimensional setting, this expectation can be shown to be confirmed if one takes a flat background as in [12]. However, the Hořava-Witten theory has only been constructed so far as an expansion in powers of the eleven dimensional gravitational coupling constant $\kappa$. The theory is only supergravity to leading order in $\kappa$: Boundary matter fields are coupled to the supergravity at higher orders. In this situation, there is no applicable result to show that the one-loop corrections should cancel amongst themselves. However, new higher order classical terms can be added to obtain cancelation at the appropriate order. This is similar to the Green-Schwartz mechanism for anomaly cancellation.

We evaluate the one-loop effective action of the various fields using $\zeta$-function regularisation. We have extended the conformal transformation technique [13] used in [11] to apply to spin 1/2 and spin 1 fields.

In the following, rather than working in the "upstairs" picture of the orbifold $S^1/Z_2$, we work in the equivalent "downstairs" description of an interval $I$. That is, we consider the space to be a manifold with a boundary, rather than a manifold with singular delta-function sources. In this way, the domain walls in the above are replaced by boundary "branes".

Our conventions are as follows: Tensor indices are $\alpha,\beta,\gamma,\ldots = 0,1,2,3,11$ in the bulk, and $\mu,\nu,\rho,\ldots = 0,1,2,3$ on the boundary. The coordinate in the direction of the interval is $x^{11}$. Capital letters $A,B,C,\ldots = 0,1,2,3,11$ index a local orthonormal frame in the bulk, while $I,J,K,\ldots = 0,1,2,3$ label a local orthonormal frame on the boundary. Conventions for the Riemann and Ricci tensors are as in Misner, Thorne and Wheeler [14]. The extrinsic curvature is defined as $K_{\mu\nu} = \nabla_\mu N_\nu$ where $N_\alpha$ is the unit *outward* pointing normal. Gamma matrices are defined by $\left\{\Gamma^\alpha, \Gamma^\beta\right\} = 2g^{\alpha\beta}$. The index $N$ denotes contraction with $N_\alpha$.

## 2. Classical background solution

The five dimensional action of heterotic $M$-theory including the $(1,1)$ moduli was derived in [4]. Here we will briefly review the field content and background solution.

The background solution consists of setting as many of the fields as possible to zero. The minimum field content consistent for describing the vacuum solution is given by the action $S^0 = S^0(\mathcal{M}) + S^0(\partial\mathcal{M})$, where the bulk and boundary actions are

$$S^0(\mathcal{M}) = -\frac{1}{\kappa_5^2}\int_{\mathcal{M}} d\mu \left(-R + G_{ij}\partial_\alpha b^i \partial^\alpha b^j + \frac{1}{2}\partial_\alpha\Phi\partial^\alpha\Phi + \frac{1}{2}e^{-2\Phi}G^{ij}\alpha_i\alpha_j\right), \tag{2.1}$$

$$S^0(\partial\mathcal{M}) = \frac{2}{\kappa_5^2}\int_{\partial\mathcal{M}^{(1)}} d\mu \left(K + \frac{\sqrt{2}}{2}\alpha_i b^i e^{-\Phi}\right) + \frac{2}{\kappa_5^2}\int_{\partial\mathcal{M}^{(2)}} d\mu \left(K - \frac{\sqrt{2}}{2}\alpha_i b^i e^{-\Phi}\right). \tag{2.2}$$

Let us first remind the reader of the origin of some of these fields. The scalar field $\Phi$ is the moduli field describing the volume of Calabi-Yau and will be referred to as the dilaton. The fields $b^i$ are the Calabi-Yau shape moduli. Indices $i,j,\ldots$ run from $1,\ldots,h^{1,1}$ and are raised and lowered by the Kahler metric $G_{ij} = -\frac{1}{2}\partial_i\partial_j \ln\mathcal{K}$, where $\mathcal{K} = d_{ijk}b^i b^j b^k$ is the Kahler potential. The constants $d_{ijk}$ are the Calabi-Yau intersection numbers. The charges $\alpha_i$ are constants. The shape moduli are constrained by $\mathcal{K} = 6$ (after the differentiation) and hence only represent $h^{1,1} - 1$ degrees of freedom.

The existence of the bulk and boundary potentials leads to a curved bulk metric and to non-trivial profiles of $\Phi$ and $b^i$ across the orbifold. We denote $y = x^{11}$. The position of the visible brane is taken to be at $y = 0$ and the hidden brane is at $y = \pi$. Upon substitution of the ansatz

$$ds^2 = a(y)^2\eta_{\mu\nu}dx^\mu dx^\nu + b(y)^2 dy^2 \tag{2.3}$$

$$\Phi = \Phi(y) \tag{2.4}$$

$$b^i = b^i(y)\,, \tag{2.5}$$

Lukas et al. have found an implicit solution to the equations of motion in terms of $h^{(1,1)}$ functions $f^i(y)$. However, they also show that by defining new constants $\bar{\alpha}^i$ and $\alpha$ by

$$d_{ijk}\bar{\alpha}^i\bar{\alpha}^j = \frac{2}{3}\alpha^i\,, \qquad \alpha = 9\left(\frac{1}{6}d_{ijk}\bar{\alpha}^i\bar{\alpha}^j\bar{\alpha}^k\right), \tag{2.6}$$



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and by using an appropriate choice of integration constants, one can find the explicit solution,

$$a = a_0 H^{1/2} \tag{2.7}$$
$$b = b_0 H^2 \tag{2.8}$$
$$\Phi = \ln\left(b_0 H^3\right) \tag{2.9}$$
$$b^i = \frac{3}{2}\frac{\alpha^i}{\alpha} \tag{2.10}$$

where

$$H = \frac{\sqrt{2}}{3}\alpha y + c_0 \,, \tag{2.11}$$

which is just the "universal" solution obtained when $h^{1,1} = 1$. In this classical solution, the fields $b^i$ are constant across the orbifold.

## 3. One-loop effective action

To evaluate the one-loop correction to the classical action for a field $\phi_a$, we perform a continuation of the lorentzian metric to a positive definite euclidean metric. We expand the euclidean action $I$ around the background fields $\phi_0^a$, i.e., $I = I(\phi_0) + I^{,a}(\phi_0)(\phi - \phi_0)_a + I^{,a}{}_b(\phi_0)(\phi - \phi_0)_a(\phi - \phi_0)^b + \ldots$. The one-loop correction to the classical action is

$$W = (-1)^f \frac{1}{2} \log\det\left(\mu_R^{-2}\Delta^a{}_b\right) \tag{3.1}$$

where we use comma (,) to denote functional differentiation with respect to the field $\phi_a$. $f = 0$ for bosons and $f = 1$ for fermions. $\Delta^a{}_b = I^{,a}{}_b$ is an operator. We define the determinant of an operator using generalised $\zeta$-functions. The $\zeta$-function is defined by a generalised trace,

$$\zeta(s) = \mathrm{tr}\left(\Delta^{-s}\right). \tag{3.2}$$

for some range of $s$ in the complex plane where the trace converges. We define the determinant of the operator by

$$\log\det\left(\mu_R^{-2}\Delta^a{}_b\right) = -\zeta'(0) - \zeta(0)\log\mu_R^2 \tag{3.3}$$

where we have used the analytic continuation of the $\zeta$-function at $s = 0$.

In curved space, the trace of an operator is often difficult to evaluate. However, we can use the properties of the effective action under conformal transformations of the metric

$$g_{\mu\nu} \to \tilde{g}_{\mu\nu} = \Omega^2 g_{\mu\nu} \,, \tag{3.4}$$

to relate the effective actions of an operator in conformally related spacetimes. We use a tilde to denote quantities calculated in the metric $\tilde{g}$. We restrict attention here to second order operators of Laplace type — that is, operators which can be written in the form

$$\Delta = -D^2 + X \,, \tag{3.5}$$

where $D_\alpha = \nabla_\alpha + iA_\alpha$ is a covariant gauge derivative. We use the $n$-bein formalism to treat fields of general spin. The covariant derivative is $\nabla_\alpha = \partial_\alpha + i\omega_\alpha{}^{AB}\Sigma_{AB}$ where $\Sigma_{AB}$ is the generator of Lorentz transformations for the appropriate representation of the Lorentz group. Under a conformal rescaling (3.4), the $n$-bein and its dual rescale as

$$\tilde{e}^A_\alpha = \Omega e^A_\alpha \,, \qquad \tilde{e}^\alpha_A = \Omega^{-1} e^\alpha_A \,, \tag{3.6}$$

while the operator (3.5) rescales to $\tilde{\Delta}$, where

$$\tilde{\Delta} = -\left(\tilde{\nabla} + i\tilde{A}\right)^2 + \tilde{X} \,. \tag{3.7}$$

If we define the conformal rescaling properties of $\tilde{X}$ and $\tilde{A}$ as

$$\tilde{X} = \Omega^{-2} X - \frac{3}{16}\left(\Omega^{-2} R - \tilde{R}\right), \qquad \tilde{A}_\alpha = A_\alpha - 2\Omega^{-1}\Omega_{;\beta}\Sigma_\alpha{}^\beta \,, \tag{3.8}$$

then the rescaled operator $\tilde{\Delta}$ is given by

$$\tilde{\Delta} = \Omega^{-7/2} \Delta \Omega^{3/2} \,. \tag{3.9}$$

Boundary conditions will also be affected by conformal transformations. If the field satisfies Robin boundary conditions

$$\left(D_N - \mathcal{S}\right)\phi = 0 \qquad \text{on } \partial\mathcal{M} \,, \tag{3.10}$$

then under a conformal rescaling, this will become

$$\left(\tilde{D}_N - \tilde{\mathcal{S}}\right)\tilde{\phi} = 0 \qquad \text{on } \partial\mathcal{M} \,. \tag{3.11}$$

since the field transforms as $\phi \to \tilde{\phi} = \Omega^{-3/2}\phi$. The boundary conditions retain the same form if we define

$$\tilde{\mathcal{S}} = \Omega^{-1}\mathcal{S} + \frac{3}{8}\left(\Omega^{-1} K - \tilde{K}\right). \tag{3.12}$$

Dirichlet boundary conditions $\phi = 0$ on $\partial\mathcal{M}$ are unchanged under conformal rescalings.

Following Dowker [13], we introduce a one parameter family of metrics which interpolate between two conformally related spacetimes. We take $\Omega = \Omega(\sigma)$. One can then show that, for operators which transform covariantly under conformal transformations,

$$W\left[\sigma = \sigma_2\right] = W\left[\sigma = \sigma_1\right] + C\left[\Omega\right], \tag{3.13}$$

where the cocycle function $C\left[\Omega\right]$ is given (in five dimensions) in terms of the generalised heat kernel coefficient $a_{5/2}(p, \Delta)$ as

$$C[\Omega] = -(-1)^f \int_{\sigma_1}^{\sigma_2} d\sigma \, a_{5/2}\left(\partial_\sigma \ln\Omega(\sigma), \tilde{\Delta}\right). \tag{3.14}$$

The $a_{5/2}\left(p, \Delta\right)$ coefficient is known for general Laplace type operators with mixed boundary conditions [15]. It is composed of geometric invariants in the metric evaluated only on the boundary of the spacetime. Hence, we can relate the effective actions of the conformally related operators in two conformally related spacetimes.

The classical background in section 2 is conformally flat in the coordinate $z$ defined by $dz = a(y)/b(y)dy$. In this new coordinate, the metric is

$$ds^2 = \left(\frac{z}{z_1}\right)^{2/5} \left(\eta_{\mu\nu}dx^\mu dx^\nu + dz^2\right), \tag{3.15}$$

where

$$z_1 = \frac{3\sqrt{2}b_0}{5\alpha a_0}. \tag{3.16}$$

The dilaton field,

$$\Phi = \frac{6}{5}\log\left(\frac{z}{z_1}\right) + \log\left(\frac{5\alpha z_1}{3\sqrt{2}}\right). \tag{3.17}$$

The values of $z$ on the two branes, $z_1$ and $z_2$ can be used as the moduli parameters of the background solution. We can use the technique described above to relate the one-loop effective action in the warped background spacetime to one in flat space.

For definiteness, we take the function $\Omega(\sigma)$ to be

$$\Omega(\sigma) = e^{(1-\sigma)\omega(z)} \tag{3.18}$$

so that flat space is at $\sigma = 0$ and the physical metric is at $\sigma = 1$. Then, the effective action is related to the flat space effective action $W_0$ by

$$W = W_0 + C[\Omega], \qquad C[\Omega] = (-1)^f \int_0^1 d\sigma\, a_{5/2}\left(\omega, \tilde{\Delta}\right). \tag{3.19}$$

From now on we use the subscript "0" to indicate a quantity calculated in the conformally transformed flat space.

## 4. The one loop effective action of the vector multiplet

### 4.1 The scalar field

There are $h^{1,1}$ scalar fields $b^i$ in the vector multiplet. However, these are constrained, and as such only represent $n_V = h^{1,1} - 1$ degrees of freedom. We can, however, write the action in terms of $n_V$ unconstrained fields $\varphi_x$. The indices $x$, $y$, etc are raised and lowered using the metric $g_{xy} = b^i_x b^j_y G_{ij}$. The action contains both bulk and boundary terms $S = S(\mathcal{M}) + S(\partial\mathcal{M})$, which are[1]

$$S(\mathcal{M}) = -\int_{\mathcal{M}} d\mu \left(\frac{1}{2}g_{xy}\partial_\alpha\varphi^x\partial^\alpha\varphi^y + V(\varphi)\right) \tag{4.1}$$

$$S(\partial\mathcal{M}) = \int_{\partial\mathcal{M}^{(1)}} d\mu U(\varphi) - \int_{\partial\mathcal{M}^{(2)}} d\mu U(\varphi). \tag{4.2}$$

The bulk potential $V(\varphi)$ and the boundary potential $U(\varphi)$ are only expressible implicitly in terms of the constrained fields $b^i$. They are

$$V(\varphi) = \frac{1}{4}e^{-2\Phi}G^{ij}\alpha_i\alpha_j, \qquad U(\varphi) = \frac{\sqrt{2}}{2}e^{-\Phi}\alpha_i b^i. \tag{4.3}$$

[1]From now on we set $k_5^2 = 1$.



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The second variation of the classical action about the background solution gives the operator

$$\Delta^x{}_y = -\nabla^2 \delta^x{}_y + V(\varphi)^{,x}{}_y \tag{4.4}$$

together with the boundary conditions

$$\left(\nabla_N \delta^x{}_y - U(\varphi)^{,x}{}_y\right) \varphi^y = 0 \qquad \text{on } \partial\mathcal{M}^{(1)}\,, \quad \left(\nabla_N \delta^x{}_y + U(\varphi)^{,x}{}_y\right) \varphi^y = 0 \qquad \text{on } \partial\mathcal{M}^{(2)}\,. \tag{4.5}$$

Although the bulk and boundary potentials are only known in terms of the constrained fields $b^i$, the second derivatives of the bulk and boundary potentials with respect to the fields $\varphi^x$ can be calculated in terms of the background scalar field $\Phi$.

$$V(\varphi)_{,xy} = \frac{4}{9}\alpha^2 e^{-2\Phi} g_{xy}\,, \qquad U(\varphi)_{,xy} = \frac{\sqrt{2}}{3}\alpha e^{-\Phi} g_{xy} \tag{4.6}$$

Since the operator has a trivial index structure in the indices $x, y$, from this point we drop this index to avoid clutter. The operator (4.4) is of Laplace type (3.5). We now perform a conformal transformation (3.4) with $\Omega = e^{\omega(z)}$. The one-loop effective action in the conformally transformed space requires the eigenvalues of the conformally transformed operator $\Delta_0 \tilde{\varphi}_n = \lambda_n \tilde{\varphi}_n$. The general solution can be found in terms of Bessel functions;

$$\tilde{\varphi}_n(z) = \sqrt{z} e^{ik_\mu x^\mu} \left(A J_{3/5}(m_n z) + B Y_{3/5}(m_n z)\right)\,, \tag{4.7}$$

where $\lambda_n = m_n^2 + k^2$. We take the position of the visible brane to be at $z_1$ and the hidden brane to be at $z_2$ ($z_2 > z_1$). We also introduce $\tau = z_1/z_2$. Applying the conformally transformed boundary condition, we obtain an implicit equation for $\mu_n = z_2 m_n$ as the roots of

$$F(m_n) = J_{2/5}(\mu_n \tau) Y_{2/5}(\mu_n) - J_{2/5}(\mu_n) Y_{2/5}(\mu_n \tau) = 0\,. \tag{4.8}$$

The $\zeta$-function is

$$\zeta(s) = n_V \int d^4x \int \frac{d^4k}{(2\pi)^4} \sum_n \left(\frac{k^2 + m_n^2}{\mu_R^2}\right)^{-s}\,. \tag{4.9}$$

The factor of $n_V$ is included since we have $n_V$ scalar fields. Performing the $k$ integrals first gives us

$$\zeta(s) = n_V \mu_R^{2s} \int d^4x \frac{\Gamma(s-2)}{(4\pi)^2 \Gamma(s)} \sum_n \mu_n^{4-2s} z_2^{2s-4}\,. \tag{4.10}$$

The evaluation of the sum over $\mu_n$ is complicated since we only know them through an implicit equation. We leave the details of the evaluation of the sum in (4.10) to appendix A. Here, we simply quote the result for the effective action of the conformally transformed operator;

$$W_0^{\rm scalar} = \frac{n_V}{(4\pi)^2} \int d^4x \left[\frac{A_{2/5}}{z_1^4} + \frac{B_{2/5}}{z_2^4} + \frac{G_{2/5}(\tau)}{z_2^4} + \frac{2781}{80000}\left(\frac{\ln(\mu_R z_1)}{z_1^4} + \frac{\ln(\mu_R z_2)}{z_2^4}\right)\right]. \tag{4.11}$$

The function $G_\nu(\tau)$ is

$$G_\nu(\tau) = \int_0^\infty dx x^3 \ln\left(1 - \frac{K_\nu(x) I_\nu(\tau x)}{I_\nu(x) K_\nu(\tau x)}\right)\,, \tag{4.12}$$

and the constants $A_\nu$ and $B_\nu$ are defined in appendix A. To obtain the full expression for the effective action, we must add the cocycle function to this result. After a simple but lengthy calculation, the details of which can be found in appendix B, the cocycle function for this operator is found to be

$$C^{\rm scalar}[\Omega] = \frac{n_V}{(4\pi)^2}\int d^4x \left[\frac{2781}{400000}\left(\frac{\ln(cz_1)}{z_1^4}+\frac{\ln(cz_2)}{z_2^4}\right)+\frac{4601}{9600000}\left(\frac{1}{z_1^4}+\frac{1}{z_2^4}\right)\right]. \quad (4.13)$$

The constant $c = 6b_0/(5\sqrt{2}\alpha a_0)$ can be absorbed into the renormalization scale $\mu_R$. After this redefinition we find the total effective action for the scalar field

$$\begin{aligned} W^{\rm scalar} &= \frac{n_V}{(4\pi)^2}\int d^4x \left[\frac{A_{2/5}}{z_1^4}+\frac{B_{2/5}}{z_2^4}+\frac{G_{2/5}(\tau)}{z_2^4}+\frac{8343}{200000}\left(\frac{\ln(\mu_R z_1)}{z_1^4}+\frac{\ln(\mu_R z_2)}{z_2^4}\right)+\right. \\ &\left. \quad + \frac{4601}{9600000}\left(\frac{1}{z_1^4}+\frac{1}{z_2^4}\right)\right]. \end{aligned} \quad (4.14)$$

### 4.2 The vector field

The five dimensional $M$-theory action contains $h^{1,1}$ U(1) gauge fields $\mathcal{A}^i_\alpha$. One of these gauge fields is the graviphoton $\mathcal{A}_\alpha$ of the gravity multiplet. This leaves $n_V$ gauge fields for the $n_V$ vector multiplets. After decomposing the $\mathcal{A}^i_\alpha$ into the graviphoton and vector multiplet gauge field by $\mathcal{A}^x_\alpha = b^x_i \mathcal{A}^i_\alpha$ and $\mathcal{A}_\alpha = \frac{2}{3} b_i \mathcal{A}^i_\alpha$, the action for the vector multiplet gauge field to quadratic order is

$$S_{\rm gauge} = \int_{\mathcal{M}} d\mu \; \frac{1}{4}\mathcal{F}^x_{AB}\mathcal{F}^{AB}_x \,. \quad (4.15)$$

Integration by parts yields

$$S_{\rm gauge} = \int_{\mathcal{M}} d\mu \; \frac{1}{2}\mathcal{A}^A_x\left(-\nabla^2\delta^B_A + R_A{}^B + \nabla_A\nabla^B\right)\mathcal{A}^x_B \quad (4.16)$$

were we have neglected boundary terms. We work in a tetrad frame. The covariant derivative contains the connection $\nabla_\alpha = \partial_\alpha + i\omega_\alpha{}^{CD}\Sigma_{CD}$ where $\left(\Sigma_C{}^D\right)_A{}^B = -\frac{i}{2}\left(\eta_{CA}\eta^{DB} - \delta^B_C\delta^D_A\right)$ is the generator of Lorenz transformations appropriate for a vector representation of the Lorenz group. Again, since the index structure in the index $x$ is trivial, we drop this index from now on.

We can add to the action a gauge fixing term $S_{\rm gf}$ and anticommuting ghost fields $c, \bar{c}$ so that $S_{\rm total} = S_{\rm gauge} + S_{\rm gf} + S_{\rm ghost}$ where

$$S_{\rm gf} = \int_{\mathcal{M}} d\mu \frac{1}{2}F^2 \,, \quad (4.17)$$

$$S_{\rm ghost} = \int_{\mathcal{M}} d\mu \bar{c}\frac{\delta F}{\delta\Lambda}c \,. \quad (4.18)$$

A convenient choice of gauge fixing term will turn out to be

$$F = \nabla\cdot\mathcal{A} + 2\omega_{;\alpha}\mathcal{A}^\alpha \,. \quad (4.19)$$

Since $\phi_{;\alpha} = -6\omega_{;\alpha}$, this gauge fixing term can also be expressed in terms of the background dilaton field. The gauge fixed action for the gauge field $S_{\rm g+gf} = S_{\rm gauge} + S_{\rm gf}$ can then be written, after an integration by parts, as

$$S_{\rm g+gf} = \int_{\mathcal{M}} d\mu\, \frac{1}{2}\mathcal{A}^A \left(-\nabla^2 \delta_A^B + R_A{}^B + 2\omega_{;A}\nabla^B - 2\omega^{;B}\nabla_A - 2\omega_{;A}{}^B + 4\omega_{;A}\omega^{;B}\right)\mathcal{A}_B\,. \quad (4.20)$$

The action can be simplified by introducing the derivative $(D_A)_B{}^C = (\nabla_A)_B{}^C + i(A_A)_B{}^C$, where the connection $A_A$ is chosen to be

$$(A_A)_B{}^C = -i\left(\omega^{;C}\eta_{AB} - \omega_{;B}\delta_A^C\right) = 2\omega_{;D}(\Sigma_A{}^D)_B{}^C\,. \quad (4.21)$$

The gauge fixed operator can now be written in the form of a Laplace-type operator, i.e.,

$$\Delta_A{}^B = -(D^2)_A{}^B + X_A{}^B\,, \quad (4.22)$$

where

$$X_A{}^B = R_A{}^B - 2\omega_{;A}{}^B + \omega_{;A}\omega^{;B} - \omega_{;C}\omega^{;C}\delta_A^B\,. \quad (4.23)$$

Boundary conditions can be found from the requirement of invariance under BRST transformations [16]. This ensures that the path integral is gauge independent. The two admissible sets of boundary conditions are known as absolute and relative. Absolute boundary conditions have

$$\mathcal{A}_N = 0\,, \qquad \left(D_N\delta_I^J + K_I{}^J\right)\mathcal{A}_J = 0\,, \qquad \nabla_N c = \nabla_N \bar{c} = 0 \qquad \text{on } \partial\mathcal{M} \quad (4.24)$$

while relative boundary conditions are obtained by setting the gauge fixing term $F = 0$ on the boundary, this leads to

$$\mathcal{A}_J = 0\,, \qquad (D_N + K + 2\omega_{;N})\,\mathcal{A}_N = 0\,, \qquad c = \bar{c} = 0 \qquad \text{on } \partial\mathcal{M}\,. \quad (4.25)$$

Although we have not written the boundary terms in the action explicitly, both these sets of boundary conditions cause the boundary terms generated by the integration by parts to vanish. The choice of boundary condition is determined by the $Z_2$ symmetry in the "upstairs" picture, which requires the tangential components $A_I$ to be odd under the $Z_2$ symmetry, and the $A_N$ to be even. In the "downstairs" picture, the boundary conditions can be derived from the boundary conditions on the eleven dimensional three form [17]. This fixes the boundary conditions to be relative — i.e., the boundary conditions are given by eq. (4.25).

We now perform a conformal transformation and calculate the eigenvalues of the conformally transformed operator. We transform the operator and the boundary conditions according to (3.8) and (3.12) respectively. The reason for our choice of gauge fixing term (4.19) should now become clear. After a conformal transformation, the connection $\tilde{A}_A = 0$.[2]

[2]If we had chosen the conventional gauge fixing term $F = \nabla\cdot\mathcal{A}$, then the conformal transformation would introduce a connection into the derivative which would couple the equations for the normal and tangential components of the gauge field. The resulting system of equations does not seem to be solvable in terms of elementary functions.

The one loop effective action can now be found from the eigenvalues of the conformally transformed operator $\Delta_0^{\mathrm{gfixed}}$. For the components of the gauge field tangential to the boundary, we find the solutions

$$\tilde{\mathcal{A}}_I = \sqrt{z} e^{ik_\mu x^\mu} (C_I J_{2/5}(m_n z) + D_I Y_{2/5}(m_n z)) \,, \tag{4.26}$$

The tangential components obey Dirichlet boundary conditions, so the implicit equation for the eigenvalues $\mu_n$ is

$$F(\mu_n) = J_{2/5}(\mu_n \tau) Y_{2/5}(\mu_n) - J_{2/5}(\mu_n) Y_{2/5}(\mu_n \tau) = 0 \,. \tag{4.27}$$

The bulk solutions for the component of the gauge field in the orbifold direction are

$$\tilde{\mathcal{A}}_{11} = \sqrt{z} e^{ik_\mu x^\mu} (C J_{3/5}(m_n z) + D Y_{3/5}(m_n z)) \,. \tag{4.28}$$

The appropriate boundary conditions are Robin and therefore leads to an implicit equation for the eigenvalues involving both Bessel functions and their derivatives. However, the particular combination in this case again reduces to (4.27).

The eigenvalues for the conformally transformed gauge field operator are the same as for the scalar field case considered in section 4.1. Hence, the effective action of the conformally transformed operator is the same as in (4.11), except here we have an additional degeneracy factor of 5 for the five degrees of freedom of the gauge field.

The operator for ghost fields $c$ and $\bar{c}$,

$$\Delta^{\mathrm{ghost}} = \frac{\delta F}{\delta \Lambda} = \nabla^2 + 2\omega_{;\alpha} \nabla^\alpha \tag{4.29}$$

is not yet in the canonical form of (3.5). However, we can introduce a connection into the derivative to get the operator into the appropriate form. We set $A_\alpha = -i\omega_{;\alpha}$, so that

$$\Delta^{\mathrm{ghost}} = \frac{\delta F}{\delta \Lambda} = (\nabla + iA)^2 - \omega_{;\alpha}\omega^{;\alpha} - \omega_{;\alpha}{}^{\alpha} \,. \tag{4.30}$$

The eigenfunctions of the conformally rescaled operator are

$$\tilde{c}(z) = z^{7/10} e^{ik_\mu x^\mu} (A J_{2/5}(m_n z) + B Y_{2/5}(m_n z)) \,. \tag{4.31}$$

The ghost fields obey Dirichlet boundary conditions, hence the implicit equation for $\mu_n$ is again

$$F(\mu_n) = J_{2/5}(\mu_n \tau) Y_{2/5}(\mu_n) - J_{2/5}(\mu_n) Y_{2/5}(\mu_n \tau) = 0. \tag{4.32}$$

The $\zeta$-function for the ghost operator follows from the scalar case in section 4.1, save for an additional degeneracy factor of 2. We can see immediately that the ghosts will cancel the contribution from two components of the gauge field, leaving us with 3 physical degrees of freedom as we would expect.

The calculation of the cocycle function turns out to be relatively simple in this gauge. More detail can be found in appendix B. We find the cocycle function to be (including the ghost fields)

$$C^{\mathrm{gauge}} = \frac{n_V}{(4\pi)^2} \int d^4x \left[ \frac{8343}{40000} \left( \frac{\ln(cz_1)}{z_1^4} + \frac{\ln(cz_2)}{z_2^4} \right) - \frac{193669}{9600000} \left( \frac{1}{z_1^4} + \frac{1}{z_2^4} \right) \right] . \tag{4.33}$$

The total effective action for the gauge fields $W^{\rm gauge} = W_0^{\rm gauge} + C^{\rm gauge}$ is

$$W^{\rm gauge} = \frac{3n_V}{(4\pi)^2}\int d^4x \left[\frac{A_{2/5}}{z_1^4} + \frac{B_{2/5}}{z_2^4} + \frac{G_{2/5}(\tau)}{z_2^4} + \frac{8343}{200000}\left(\frac{\ln(\mu_R z_1)}{z_1^4} + \frac{\ln(\mu_R z_2)}{z_2^4}\right) - \right.$$
$$\left. - \frac{193669}{28800000}\left(\frac{1}{z_1^4} + \frac{1}{z_2^4}\right)\right]. \tag{4.34}$$

We have checked the gauge independence of this result by calculating various quantities in different gauges. Firstly, we can evaluate $\zeta(0)$ via the $a_{5/2}(1,\Delta)$ heat kernel coefficient using the Lorentz gauge $F = \nabla \cdot \mathcal{A}$. We have also performed the calculation of the flat space effective action in an axial-type gauge with gauge fixing term $F = \omega_{;A}\mathcal{A}^A$. In all three gauges we find exact agreement between the various quantities.

### 4.3 The fermion field

The vector multiplet contains $n_V$ pairs of symplectic real fermions $\lambda_a^x$. The symplectic index $a = 1, 2$ is raised and lowered using an antisymmetric metric $\epsilon_{ab}$. The fermion action in the vector multiplet is

$$S_{\rm fermion} = \int_{\mathcal{M}} d\mu \,\left(\frac{1}{2}\bar{\lambda}^{a\,x}\gamma^\alpha\nabla_\alpha\lambda_{a\,x} - \frac{1}{2}M_a{}^b{}_x{}^y\bar{\lambda}^{a\,x}\lambda_{b\,y}\right). \tag{4.35}$$

The mass matrix $M_a{}^b{}_x{}^y$ depends on the background dilaton field. We use the fermion mass term for gauged supergravity models [18]. (We believe that the mass term quoted in [4] is incorrect). Explicitly, in this background,

$$M^a{}_{b\,xy} = -\alpha\frac{\sqrt{2}}{12}e^{-\Phi}(\sigma_3)^a{}_b g_{xy} \tag{4.36}$$

where $(\sigma_3)^a{}_b = {\rm diag}(1,-1)$ is the Pauli matrix. Again, the operator is diagonal in indices $x$, so we leave them out from now on for simplicity. Boundary conditions on the fermions are

$$\gamma^{11}\lambda^a = (\sigma_3)^a{}_b\lambda^b \qquad \text{on } \partial\mathcal{M}\,. \tag{4.37}$$

We define a column spinor

$$\Lambda = \begin{pmatrix}\lambda^1\\ \lambda^2\end{pmatrix} \tag{4.38}$$

and matrices

$$\Gamma_A = \begin{pmatrix}\gamma_A & 0\\ 0 & -\gamma_A\end{pmatrix}. \tag{4.39}$$

We can now write the action as

$$S_{\rm fermion} = \int_{\mathcal{M}} d\mu\;\bar{\Lambda}\left(\Gamma^A\nabla_A - m\right)\Lambda\,, \tag{4.40}$$

where $m = -\alpha\frac{\sqrt{2}}{12}e^{-\Phi}$ and we have defined $\bar{\Lambda} = \Lambda^\dagger\Gamma^0$. Introducing the projection operators

$$P_\pm = \frac{1}{2}(1 \pm \Gamma^{11})\,, \tag{4.41}$$



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the boundary conditions on the fermions (4.37) can be written

$$P_-\chi = 0 \qquad \text{on } \partial\mathcal{M}\,. \tag{4.42}$$

The Dirac equation $\mathcal{D} = \not\nabla - m$ is a first order differential equation and hence not of Laplace type (3.5). However, we can form a second order differential operator from the square of the Dirac operator. We make two copies of the spin bundle with opposite orientations, $S^+$ and $S^-$. The Dirac operator maps spinors in $S^+$, denoted $\Lambda$, to image spinors in $S^-$, denoted $\Lambda'$. The adjoint operator is $\mathcal{D}^* = -\not\nabla - m$. The eigenvalue equation is then

$$\mathcal{D}\Lambda = \eta\Lambda'\,, \qquad \mathcal{D}^*\Lambda' = \eta\Lambda\,, \tag{4.43}$$

with the boundary conditions

$$P_-\Lambda = 0\,, \qquad P_+\Lambda' = 0 \qquad \text{on } \partial\mathcal{M}\,. \tag{4.44}$$

Provided $\eta \neq 0$, we can form a second order operator with eigenvalue $\eta^2$. One choice of second order operator is

$$\Delta = \mathcal{D}^*\mathcal{D} = -\nabla^2 + \frac{1}{4}R + m^2 + \not\partial m \tag{4.45}$$

where we have allowed for the possibility that $m$ is non-constant across the orbifold. The operator $\Delta$ maps spinors in $S^+$ to $S^+$. The boundary conditions on the fermions are

$$P_-\Lambda = 0\,, \qquad P_+\mathcal{D}\Lambda = 0\,, \qquad \text{on } \partial\mathcal{M}\,. \tag{4.46}$$

There is also an alternative squared operator

$$\Delta^* = \mathcal{D}\mathcal{D}^* = -\nabla^2 + \frac{1}{4}R + m^2 - \not\partial m \tag{4.47}$$

which maps spinors in $S^-$ to $S^-$. The boundary conditions are

$$P_+\Lambda = 0\,, \qquad P_-\mathcal{D}^*\Lambda = 0\,, \qquad \text{on } \partial\mathcal{M}\,. \tag{4.48}$$

The spectrum of (4.45) and (4.47) are identical if there are no zero modes.

The massless Dirac operator transforms covariantly under the conformal transformation (3.4),

$$\tilde{\not\nabla} = \Omega^{-3}\,\not\nabla\,\Omega^2\,. \tag{4.49}$$

The presence of a mass breaks the conformal invariance. We can, however, introduce a mass that scales under the conformal transformation, such that $\tilde{m} = \Omega^{-1}m$. Then both $\mathcal{D}$ and $\mathcal{D}^*$ transform covariantly under conformal transformations. The boundary conditions (4.46) and (4.48) are invariant under conformal transformations. The squared Dirac operator does not transform covariantly. However, in appendix B it is shown that the effective action for fermions can be written in terms of the effective action in the conformally related space using

$$W = W_0 - \int_0^1 d\sigma\ \frac{1}{2}\left(a_{5/2}(\omega,\Delta) + a_{5/2}(\omega,\Delta^*)\right). \tag{4.50}$$

As we have mentioned, if there are no zero modes, then the spectrum of $\Delta$ and $\Delta^*$ are equivalent and $a_{5/2}(\omega,\Delta) = a_{5/2}(\omega,\Delta^*)$.



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We are therefore lead to consider the eigenvalues of the operator $\Delta_0 = \mathcal{D}_0^* \mathcal{D}_0$ in flat space. In this case, the bulk solutions are again related to the Bessel functions. Defining integration constants $A^{(\pm)}$ and $B^{(\pm)}$ such that $\Gamma^{11} A^{(\pm)} = \pm A^{(\pm)}$ and $\Gamma^{11} B^{(\pm)} = \pm B^{(\pm)}$, we can write the solution as

$$\tilde{\Lambda}_n(z) = \sqrt{z}\left(A^{(-)} J_{2/5}(m_n z) + B^{(-)} Y_{2/5}(m_n z) + A^{(+)} J_{3/5}(m_n z) + B^{(+)} Y_{3/5}(m_n z)\right). \tag{4.51}$$

The boundary conditions (4.46) in the conformally transformed space can be written as

$$P_- \tilde{\Lambda} = 0\,, \qquad (\partial_z - m_0)\, P_+ \tilde{\Lambda} = 0\,, \qquad \text{on } \partial\mathcal{M}\,. \tag{4.52}$$

This gives us the implicit equation for the eigenvalues $\mu_n$;

$$F(\mu_n) = J_{2/5}(\mu_n \tau) Y_{2/5}(\mu_n) - J_{2/5}(\mu_n) Y_{2/5}(\mu_n \tau) = 0\,. \tag{4.53}$$

The fermion $\zeta$-function is

$$\zeta(s) = 4 n_V \mu_R^{-2s} \int d^4x \frac{\Gamma(s-2)}{(4\pi)^2 \Gamma(s)} \sum_n \mu_n^{4-2s} z_2^{2s-4}\,. \tag{4.54}$$

Here, the degeneracy factor of 4 is equal to the number of degrees of freedom of the fermion field. The sum over eigenvalues can be evaluated using results from appendix A to give

$$W_0^{\text{fermion}} = -n_V \frac{4}{(4\pi)^2} \int d^4x \left[\frac{A_{2/5}}{z_1^4} + \frac{B_{2/5}}{z_2^4} + \frac{G_{2/5}(\tau)}{z_2^4} + \frac{2781}{80000}\left(\frac{\ln(\mu_R z_1)}{z_1^4} + \frac{\ln(\mu_R z_2)}{z_2^4}\right)\right]. \tag{4.55}$$

The cocycle function for this operator can be calculated using results from appendix B to be

$$C^{\text{fermion}}[\Omega] = -4 \frac{n_V}{(4\pi)^2} \int d^4x \left[\frac{2781}{400000}\left(\frac{\ln(c z_1)}{z_1^4} + \frac{\ln(c z_2)}{z_2^4}\right) - \frac{18129}{3200000}\left(\frac{1}{z_1^4} + \frac{1}{z_2^4}\right)\right]. \tag{4.56}$$

The effective action for the fermion in the vector multiplet is then

$$\begin{aligned} W^{\text{fermion}} = -n_V \frac{4}{(4\pi)^2} \int d^4x \Bigg[&\frac{A_{2/5}}{z_1^4} + \frac{B_{2/5}}{z_2^4} + \frac{G_{2/5}(\tau)}{z_2^4} + \frac{8343}{200000}\left(\frac{\ln(\mu_R z_1)}{z_1^4} + \frac{\ln(\mu_R z_2)}{z_2^4}\right) - \\ &- \frac{18129}{3200000}\left(\frac{1}{z_1^4} + \frac{1}{z_2^4}\right)\Bigg]. \end{aligned} \tag{4.57}$$

### 4.4 Total effective action

The effective actions for all the fields in the vector multiplets contain the function $G_\nu(\tau)$. We can approximate this function for small brane separation $\tau \to 1$ using the asymptotic expansion of the Bessel function as [11]

$$z_2^{-4} G_\nu(\tau) \approx -\frac{3}{8} \frac{\zeta_R(5)}{(z_1 - z_2)^4}\,, \qquad 1 - \tau \ll 1\,. \tag{4.58}$$

Hence, the effective actions of the individual fields diverge as the brane separation goes to zero.

We can see from the results in section 4.1, 4.2 and 4.3 that the contributions to the effective action from the flat space effective potential $W_0$ will cancel between the bosonic and fermionic sectors of the multiplet. The logarithmic terms in the cocycle functions also cancel. There is, however, a piece left over from the cocycle function, taking the form

$$W^{\text{total}} = W^{\text{scalar}} + W^{\text{gauge}} + W^{\text{fermion}} = \frac{n_V}{(4\pi)^2}\int d^4x \frac{89}{30000}\left(\frac{1}{z_1^4}+\frac{1}{z_2^4}\right). \tag{4.59}$$

Although the individual bosonic and fermionic contributions to the effective potential diverge when the separation of the two branes is reduced to zero, the finite residual term in the total effective action remains finite. From (3.16), it can be seen that the residual term in the effective action is of order $\alpha^4$ in the brane charge expansion.

In the low energy effective four dimensional theory the moduli $z_1$ and $z_2$ contribute to the dilaton $S$ and axion $T$ moduli fields and we would not expect to find an explicit superpotential unless the supersymmetry was broken. One way in which the apparent contradiction with our result may be resolved would be through higher derivative terms in the Hořava-Witten model. Curvature terms related to string corrections have been discussed in [19]. As an example, consider

$$S_{R^5} = \frac{1}{16\pi\kappa^2}\left(\frac{\kappa^{2/3}}{4\pi}\right)^3 \int_{\partial\mathcal{M}} \sqrt{g} d^{10}x\, I_3(R) I_2(R)\,, \tag{4.60}$$

where $I_3(R)$ is the cubic combination of curvature terms which appear in the six dimensional Gauss-Bonnet identity and $I_2(R)$ is any quadratic combination of curvature tensors. When reduced to five dimensions, the action would contain a term

$$S_{R^2} = \chi \int_{\partial\mathcal{M}} \sqrt{g} d^4x I_2(R) \tag{4.61}$$

where $\chi$ is the Euler number of the Calabi-Yau manifold used in the reduction to five dimensions. Since $\chi = 2(h^{1,1} - h^{1,2})$, this term has the correct dependence on $n_V$, $z_1$ and $z_2$ to be able to cancel the one loop term from the vector multiplets.

## 5. Conclusions

We have calculated the one-loop effective action for the vector multiplets related to the $h^{1,1}$ moduli of the Calabi-Yau compactification of heterotic $M$-theory. We find that effective action of the individual fields induces a potential for the moduli which diverges as the brane separation is reduced. These terms cancel between the fermions and bosons in the vector multiplet. However, boundary terms do not cancel exactly. These terms are of order $\kappa_5^2$ relative to the bulk gravitational action, and hence of higher order than the validity of the original $M$-theory action. We propose that these boundary terms could be cancelled by higher derivative terms in the $M$-theory action. If this is the case, then this term should be taken into account in future work on the one-loop effective action in heterotic $M$-theory. For, example, it would be interesting to consider the effect of supersymmetry breaking by gaugino condensation [20, 21] on the quantum effective action. Since supersymmetry is broken by the topology of the orbifold direction, one would expect that the casimir effect would be non-zero and give an extra contribution to the classical moduli potential induced by the gaugino condensate.


## Acknowledgments

We are grateful to James Gray for discussions about heterotic $M$-theory.


## A. Evaluation of the $\zeta$-function

The $\zeta$-function is

$$\zeta(s) = \int d^4x \int \frac{d^4k}{(2\pi)^4} \sum_n \left( \frac{k^2 + m_n^2}{\mu_R^2} \right)^{-s} . \tag{A.1}$$

Performing the momentum integrals gives us

$$\zeta(s) = \mu_R^{2s} \int d^4x \frac{\Gamma(s-2)}{(4\pi)^2 \Gamma(s)} \sum_n \mu_n^{4-2s} z_2^{2s-4} . \tag{A.2}$$

The summation of the masses is complicated since we only know the masses as implicit solutions of an equation $F(\mu_n) = 0$. However, we can follow [22, 23, 24] and use the residue theorem to write the sum as a contour integral, that is

$$\hat{\zeta}(2s-4) = \sum_n \mu_n^{4-2s} = \int_{\mathcal{C}} dz\; z^{4-2s} \frac{d}{dz} \ln |F(z)| . \tag{A.3}$$

Here the contour $\mathcal{C}$ is any contour which encloses the positive zeros of $F(z)$. For the implicit eigenvalue equation (4.8), we must restrict $s$ to lie in the range $5/2 < s < 3$. We chose to work with the contour shown in figure 1. For $s > \frac{5}{2}$, the contribution to the integral from the large semi circle vanish, and we are left with the contribution along the imaginary axis and the small semi circle.

This results in

$$\hat{\zeta}(2s-4) = \frac{\sin(\pi s)}{\pi} \int_\epsilon^\infty dx\; x^{4-2s} \frac{d}{dx} \ln |P(x)| + \int_{\mathcal{C}_\epsilon} \frac{dz}{2\pi i} z^{4-2s} \frac{d}{dz} \ln |F(z)| , \tag{A.4}$$

where we have defined $P(x) = F(ix)$. Explicitly, we take (up to an irrelevant multiplicative constant),

$$P(x) = I_\nu(\tau x) K_\nu(x) - I_\nu(x) K_\nu(\tau x) . \tag{A.5}$$

We also define

$$P_a(x) = I_\nu(x) K_\nu(\tau x) . \tag{A.6}$$

We can now write

$$\int_\epsilon^\infty dx\, x^{4-2s} \frac{d}{dx} \ln |P(x)| = \int_\epsilon^\infty dx\, x^{4-2s} \frac{d}{dx} \ln \left| \frac{P(x)}{P_a(x)} \right| + \int_\epsilon^\infty dx\, x^{4-2s} \frac{d}{dx} \ln |P_a(x)| . \tag{A.7}$$



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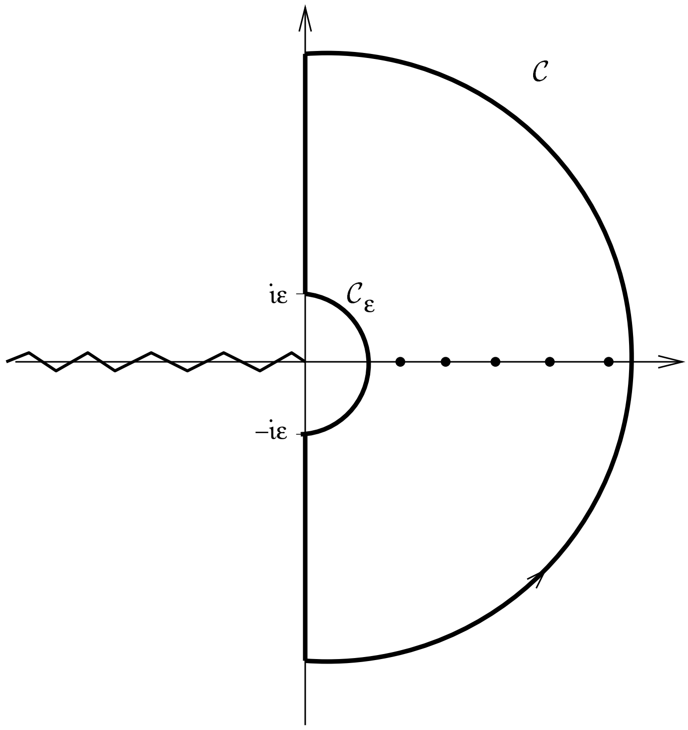


**Figure 1:** Contour used for the contour integral in (A.3).

The first term on the r.h.s. of (A.7) is now analytic at $s = 0$. The only obstruction to analytically continuing to $s = 0$ is the second term. We can write

$$I_\nu(x) = \frac{e^x}{\sqrt{2\pi x}}\Sigma^I_\nu(x)\,, \qquad K_\nu(x) = \sqrt{\frac{\pi}{2x}}e^{-x}\Sigma^K_\nu(x)\,. \tag{A.8}$$

Using the asymptotic expansion of Bessel functions, we can define constants $\beta_n$ by

$$\ln\left|\Sigma^I_\nu\right| = \sum_{n=1}^{\infty}\beta_n x^{-n}\,. \tag{A.9}$$

Explicit expressions for the $\beta_n$ can be found in [8]. We also have that

$$\Sigma^I_\nu(x) \simeq \Sigma^K_\nu(-x)\,. \tag{A.10}$$

We define regularised functions by subtracting off the terms which cause the integrand to diverge at large $x$,

$$\mathcal{U}_I(x) = \frac{d}{dx}\ln\left|\Sigma^I_\nu(x)\right| + \sum_{n=0}^{3} n\beta_n x^{-n-1} + 4\beta_4 x^{-5}e^{-k/x} \tag{A.11a}$$

$$\mathcal{U}_K(x) = \frac{d}{dx}\ln\left|\Sigma^K_\nu(x)\right| + \sum_{n=0}^{3}(-1)^n n\beta_n x^{-n-1} + 4\beta_4 x^{-5}e^{-k/x}\,. \tag{A.11b}$$

We can now write the second term on the r.h.s. of (A.7) using A.11 , A.9 and A.10. The integrals of the terms which we have subtracted off can be performed for $s > 1$. We can now analytically continue to $s < \frac{1}{2}$ and take the limit $\epsilon = 0$. This gives

$$\hat{\zeta}(2s-4) = -\frac{4\sin\pi s}{\pi}\left\{g_\nu(s) + b_\nu(s) + a_\nu(s)\tau^{2s-4} + \beta_4 k^{-2s}\Gamma(2s)(1+\tau^{2s-4})\right\}, \quad \text{(A.12)}$$

where we have defined the functions

$$g_\nu(s) = -\frac{1}{4}\int_0^\infty dx\ x^{4-2s}\frac{d}{dx}\ln\left|\frac{P(x)}{P_a(x)}\right|, \quad \text{(A.13)}$$

$$b_\nu(s) = -\frac{1}{4}\int_0^\infty dx\ x^{4-2s}\mathcal{U}_I(x) \quad \text{(A.14)}$$

and

$$a_\nu(s) = -\frac{1}{4}\int_0^\infty dx\ x^{4-2s}\mathcal{U}_K(x)\,. \quad \text{(A.15)}$$

At $s = 0$, we have, after a redefinition of the renormalization constant $\mu_R$,

$$\zeta(0) = -\frac{\beta_4}{16\pi^2}\left(z_1^{-4} + z_1^{-4}\right) \quad \text{(A.16)}$$

$$\zeta'(0) = -\frac{1}{8\pi^2}\left(\frac{G_\nu}{z_2^4} + \frac{B_\nu}{z_2^4} + \frac{A_\nu}{z_1^4}\right) - \frac{\beta_4}{8\pi^2}\left(\frac{\ln z_1\mu_R}{z_1^4} + \frac{\ln z_2\mu_R}{z_2^4}\right), \quad \text{(A.17)}$$

where $B_\nu = b_\nu(0)$, $A_\nu = a_\nu(0)$, and $G_\nu = g_\nu(0)$.

## B. Cocycle functions for spin 0,1/2 and 1

We begin by reviewing the relationship between the cocycle functions and the heat kernel coefficients. Suppose that $\Delta(\sigma)$ is a one parameter family of operators

$$\Delta(\sigma) = e^{-2\sigma\omega}\Delta_0\,. \quad \text{(B.1)}$$

Consider the derivative of the generalised $\zeta$ function

$$\partial_\sigma\zeta(s) = -2s\,\mathrm{tr}(\Delta^{-s}\omega)\,. \quad \text{(B.2)}$$

We denote this trace by $\zeta(s,\omega]$. The derivative of the effective action

$$\partial_\sigma W = \zeta(0,\omega] = a_{5/2}(\omega,\Delta) \quad \text{(B.3)}$$

where $a_{5/2}(\omega,\Delta)$ is a heat kernel coefficient. Integrating this with respect to $\sigma$ gives equation (3.14).

In the case of fermions, we have an operator $\Delta = \mathcal{D}^*\mathcal{D}$ where

$$\mathcal{D} \equiv \mathcal{D}(\sigma) = e^{-3\sigma\omega}\mathcal{D}_0 e^{2\sigma\omega}\,. \quad \text{(B.4)}$$

This time

$$\partial_\sigma\Delta(\sigma) = -\omega\mathcal{D}^*\mathcal{D} - \mathcal{D}^*\omega\mathcal{D} - 2[\omega,\Delta]\,. \quad \text{(B.5)}$$

Using the cyclic property of the trace, this results in

$$\partial_\sigma\zeta(s) = -s\,\mathrm{tr}(\Delta^{-s}\omega) - s\,\mathrm{tr}(\mathcal{D}\Delta^{-1-s}\mathcal{D}^*\omega)\,. \quad \text{(B.6)}$$

Note that $\mathcal{D}\Delta^n\mathcal{D}^* = \Delta^{*n+1}$, and using the definition of $\Delta^{-s}$ from [25, page 78] we now find

$$\partial_\sigma W = -\frac{1}{2}\left(a_{5/2}(\omega,\Delta) + a_{5/2}(\omega,\Delta^*)\right). \tag{B.7}$$

Following the steps above now gives equation (4.50).

The $a_{5/2}$ heat kernel coefficient is known for general Laplace type operators of the form

$$\Delta = -D^2 + X \tag{B.8}$$

where $D = \nabla + iA$ [15]. Its explicit form for a particular operator on a particular background is quite lengthy. In this appendix we show some more detail in the calculation of the cocycle function for spin 0,1/2 and 1 fields in five dimensions for the case of a space which is conformal to flat space with a flat boundary, which will result in expression which are much more manageable that the full general coefficient.

The field is split up into components obeying Dirichlet boundary conditions and components obeying Robin type boundary conditions by projection operators, such that

$$P_-\phi = 0\,, \qquad (D_N - \mathcal{S})P_+\phi = 0 \qquad \text{on } \partial\mathcal{M}\,, \tag{B.9}$$

where the projection operators obey $P_+ + P_- = 1$, $P_\pm P_\pm = P_\pm$ and $P_\pm P_\mp = 0$. The combination $\chi = P_+ - P_-$. The field strength of the derivative is defined as $\Omega_{AB} = [D_A, D_B]$. Tangential derivatives with respect to the Levi-Civita connection of the boundary are denoted by "|" and contain the spin connection and the connection $A$. Since the boundary is intrinsicly flat, this means $E_{|I} = \partial_I E + i\omega_I{}^{CD}\Sigma_{CD}E + iA_I E$. For example, if $D$ is the covariant derivative for spin 1/2 fields, then $\Gamma^J_{|I} = -K_I{}^J\Gamma^N$.

The $a_{5/2}(f,\Delta)$ heat kernel coefficient can be written as

$$a_{5/2}(f,\Delta) = \frac{1}{5760}(4\pi)^{-2}\,\mathrm{tr}\int_{\partial\mathcal{M}} d\mu\Big\{fa^{(1)} + f_{;N}a^{(2)} + f_{;NN}a^{(3)} + f_{;NNN}a^{(4)} + f_{;NNNN}a^{(5)}\Big\} \tag{B.10}$$

where in five dimensions with a four dimensional *maximally symmetric* boundary, the $a^{(n)}$ simplify to

$$\begin{aligned}
a^{(1)} = {} & -360\chi X_{;NN} + 1440\mathcal{S}X_{;N} + 720\chi X^2 - 240\chi XR + \\
& +48\chi R_{;AA} + 20\chi R^2 - 8\chi R_{AB}R_{AB} + 8\chi R_{ABCD}R_{ABCD} + \\
& +120\chi XR_{NN} - 20\chi RR_{NN} + 480\mathcal{S}^2R + 12\chi R_{;NN} + 15\chi R_{NN;NN} - \\
& -270\mathcal{S}R_{;N} + 120R_{NN}\mathcal{S}^2 + 960\mathcal{S}\mathcal{S}_{|II} + 16\chi R_{INJN}R_{IJ} - 17\chi R_{NN}R_{NN} - \\
& -10\chi R_{INJN}R_{INJN} - 2880X\mathcal{S}^2 + 1440\mathcal{S}^4 + (90P_+ + 450P_-)\,X_{;N}K - \\
& -\left(\frac{111}{2}P_+ + 42P_-\right)KR_{;N} - \frac{15}{2}P_+KR_{NN;N} - 1440K\mathcal{S}X + 90\mathcal{S}KR_{NN} + \\
& +225\mathcal{S}KR + \left(-\frac{405}{2}P_+ + \frac{135}{2}P_-\right)XK^2 + \\
& +\left(\frac{73}{2}P_+ - \frac{71}{4}P_-\right)K^2R + \left(-\frac{313}{16}P_+ + \frac{433}{16}P_-\right)K^2R_{NN} +
\end{aligned}$$

$$
\begin{aligned}
&+270\mathcal{S}K^3+1170\mathcal{S}^2K^2+2160\mathcal{S}^3K+\left(\frac{5619}{256}P_+ +\frac{51}{256}P_-\right)K^4-\\
&-180X^2+180\chi X\chi X-120\mathcal{S}_{|I}\mathcal{S}_{|I}+720\chi\mathcal{S}_{|I}\mathcal{S}_{|I}-\\
&-\frac{105}{4}\Omega_{IJ}\Omega_{IJ}+120\chi\Omega_{IJ}\Omega_{IJ}+\frac{105}{4}\chi\Omega_{IJ}\chi\Omega_{IJ}-45\Omega_{IN}\Omega_{IN}+\\
&+180\chi\Omega_{IN}\Omega_{IN}-45\chi\Omega_{IN}\chi\Omega_{IN}+360\left(\Omega_{IN}\mathcal{S}_{|I}\chi-\Omega_{IN}\chi\mathcal{S}_{|I}\right)-\\
&-90\chi\chi_{|I}\Omega_{IN}-180\chi_{|I}\chi_{|J}\Omega_{IJ}+90\chi\chi_{|I}\chi_{|J}\Omega_{IJ}+90\chi\chi_{|I}\Omega_{IN;N}+\\
&+120\chi\chi_{|I}\Omega_{IJ|J}-300\chi_{|I}X_{|I}+180\chi_{|I}\chi_{|I}X+90\chi\chi_{|I}\chi_{|I}X-\\
&-240\chi_{|II}X-30\chi_{|I}\chi_{|I}R-60\chi_{|I}\chi_{|J}R_{IJ}+30\chi_{|I}\chi_{|J}R_{INJN}-\\
&-\frac{1485}{32}\chi_{|I}\chi_{|J}K^2-405\chi_{|I}\mathcal{S}_{|I}K+\frac{15}{4}\chi_{|I}\chi_{|I}\chi_{|J}\chi_{|J}+\frac{15}{8}\chi_{|I}\chi_{|J}\chi_{|I}\chi_{|J}-\\
&-\frac{15}{4}\chi_{|II}\chi_{|JJ}-\frac{105}{2}\chi_{|IJ}\chi_{|IJ}-15\chi_{|I}\chi_{|I}\chi_{|JJ}-\frac{135}{2}\chi_{|J}\chi_{|IIJ}\\
a^{(2)} &= \left(\frac{195}{2}P_+-60P_-\right)R_{;N}-240R\mathcal{S}+90R_{NN}\mathcal{S}-270\mathcal{S}_{|II}-\\
&-\left(630P_+-450P_-\right)X_{;N}+1440X\mathcal{S}-720\mathcal{S}^3+\\
&+\left(90P_++450P_-\right)KX+\left(\frac{555}{16}P_+-\frac{75}{16}P_-\right)KR_{NN}-\left(\frac{75}{4}P_++\frac{135}{2}P_-\right)KR-\\
&-600K\mathcal{S}^2-\frac{1335}{8}K^2\mathcal{S}-\left(\frac{969}{64}P_+-\frac{45}{64}P_-\right)K^3+\\
&+210\chi_{|I}\mathcal{S}_{|I}+\frac{735}{32}\chi_{|I}\chi_{|I}K+135\chi\chi_{|I}\Omega_{IN}\\
a^{(3)} &= 60\chi R-90\chi R_{NN}-360\chi X+360\mathcal{S}^2+\\
&+30K\mathcal{S}-\left(\frac{15}{32}P_++\frac{1485}{32}P_-\right)K^2-30\chi_{|I}\chi_{|I}\\
a^{(4)} &= -180\mathcal{S}+\left(30P_+-105P_-\right)K\\
a^{(5)} &= 45\chi\,. \qquad \text{(B.11)}
\end{aligned}
$$

The cocycle function for a conformally flat five dimensional metric

$$ds^2=e^{-2\omega(z)}\left(dx^\mu dx^\nu\eta^{(4)}_{\mu\nu}+dz^2\right) \qquad \text{(B.12)}$$

has the general form

$$C[\Omega]=\frac{1}{5760}(4\pi)^{-2}\int_{\partial\mathcal{M}}d\mu\,(\omega A+B)\,, \qquad \text{(B.13)}$$

where

$$A=c_1\omega''''+c_2\omega'''\omega'+c_3\omega''^2+c_4\omega''\omega'^2+c_5\omega'^4\,, \qquad \text{(B.14)}$$

$$B=d_1\omega''''+d_2\omega'''\omega'+d_3\omega''^2+d_4\omega''\omega'^2+d_5\omega'^4\,. \qquad \text{(B.15)}$$

We have used a prime ($'$) to represent derivatives with respect to the conformal coordinate $z$. In the following section we will calculate the coefficients $c_n$ and $d_n$ for scalar, vector and fermion fields in five dimensional conformally flat metrics with maximally symmetric boundaries.

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### B.1 Scalar fields

We restrict attention to operators with $X = \xi R$. The scalar field in the vector multiplet considered in section 4.1 has this form with $\xi = 2/7$.

#### B.1.1 Dirichlet boundary conditions

For Dirichlet boundary conditions, the field $\phi$ vanishes on the boundary,

$$\phi = 0 \qquad \text{on } \partial\mathcal{M}\,. \tag{B.16}$$

The projection operators are therefore simply

$$P_- = 1\,, \qquad P_+ = 0\,, \qquad \chi = P_+ - P_- = -1\,. \tag{B.17}$$

Tangential derivatives of the projection operators are zero as is the field strength of the connection,

$$\chi_{|I} = 0\,, \qquad \chi_{|IJ} = 0\,, \qquad \chi_{|IJK} = 0\,, \qquad \Omega_{AB} = 0\,. \tag{B.18}$$

The cocycle function for scalar fields with Dirichlet boundary conditions in this geometry was calculated in [11] for the special case of when $\omega = -\beta \ln z$. We find complete agreement with their results. The coefficients $c_i$ and $d_i$ are listed in table 1.

#### B.1.2 Robin boundary conditions

Robin boundary conditions have some combination of the field and its derivative equal to zero on the boundary,

$$(D_N - \mathcal{S})\,\phi = 0 \qquad \text{on } \partial\mathcal{M}\,. \tag{B.19}$$

The projection operators are therefore

$$P_- = 0 \qquad P_+ = 1 \qquad \chi = P_+ - P_- = 1\,. \tag{B.20}$$

Again, tangential derivatives of the projection operators are zero, as is the commutator of two derivatives

We restrict our attention to boundary conditions with $\mathcal{S} = -\eta K$, where $K$ is the trace of the extrinsic curvature of the boundary. The scalar field in the vector multiplet we consider in section 4.1 is of this form with $\eta = 1/2$. We present the coefficients $c_i$ and $d_i$ in table 1.

### B.2 Vector fields

The gauge fixed operator in the gauge $F = \nabla \cdot \mathcal{A} + 2\omega_{;A}\mathcal{A}^{;A}$ is of Laplace type with

$$(A_A)_B^{\ C} = 2\omega_{;D}(\Sigma_A^{\ D})_B^{\ C}\,, \tag{B.21}$$

and

$$X_A^{\ B} = R_A^{\ B} - 2\omega_{;A}^{\ \ \ B} + \omega_{;A}\omega^{;B} - \omega_{;C}\omega^{;C}\delta_A^B\,. \tag{B.22}$$

We define the projection operators

$$P_- = \delta_B^A - N^A N_B\,, \qquad P_+ = N^A N_B\,, \qquad \chi = P_+ - P_- = \delta_B^A - 2N^A N_B \tag{B.23}$$

| Field | $c_1$ | $c_2$ | $c_3$ | $c_4$ | $c_5$ |
|---|---|---|---|---|---|
| Dirichlet Scalar | $2880\hat{\xi}$ | $-8640\hat{\xi}$ | $-8640\hat{\xi}$<br>$-46080\hat{\xi}^2$ | $138240\hat{\xi}^2$ | $-103680\hat{\xi}^2$ |
| Robin Scalar | $-2880\hat{\xi}$ | $8640\hat{\xi}$<br>$+46080\hat{\xi}\hat{\eta}$ | $8640\hat{\xi}$<br>$+46080\hat{\xi}^2$ | $-138240\hat{\xi}^2$<br>$-368640\hat{\xi}\hat{\eta}^2$<br>$-138240\hat{\xi}\hat{\eta}$ | $103680\hat{\xi}^2$<br>$+552960\hat{\eta}^2\hat{\xi}$<br>$+368640\hat{\eta}^4$ |
| Vector | $-540$ | $540$ | $0$ | $540$ | $-135$ |
| Fermion | $+1440b$ | $-2880b^2$ | $0$ | $-5760b^3$ | $2880b^4$ |
| Field | $d_1$ | $d_2$ | $d_3$ | $d_4$ | $d_5$ |
| Dirichlet Scalar | $-45$ | $150$<br>$+3600\hat{\xi}$ | $120$<br>$+2880\hat{\xi}$ | $-\frac{135}{2}$<br>$-17280\hat{\xi}$ | $-\frac{165}{8}$<br>$+3240\hat{\xi}$ |
| Robin Scalar | $45$ | $-120$<br>$-720\hat{\eta}$<br>$-5040\hat{\xi}$ | $-120$<br>$-2880\hat{\xi}$ | $\frac{75}{2}$<br>$+20160\hat{\xi}$<br>$+1680\hat{\eta}$<br>$+46080\hat{\eta}\hat{\xi}$<br>$+5760\hat{\eta}^2$ | $\frac{3}{8}$<br>$-1080\hat{\xi}$<br>$-120\hat{\eta}$<br>$-69120\hat{\eta}\hat{\xi}$<br>$-3840\hat{\eta}^2$<br>$-46080\hat{\eta}^3$ |
| Vector | $-45$ | $-675$ | $-420$ | $\frac{2205}{2}$ | $-\frac{1107}{8}$ |
| Fermion | $0$ | $120$<br>$+2520b$ | $-1440b$ | $-60$<br>$-480b$<br>$-2880b^2$ | $-\frac{99}{2}$<br>$-60b$<br>$-240b^2$<br>$-1440b^3$ |

**Table 1:** Coefficients for cocycle function in five dimensions for flat boundaries. The vector field result includes the ghost fields, and the fermion result includes the fermion sign factor and the factor of 4 for the number of degrees of freedom of the fermion. The scalar mass squared is $\xi R$, the Robin boundary operator $(\partial_N + \eta K)$ and the fermion mass $m = b\omega' e^{\omega}$. The parameters $b$, $\hat{\xi} = \xi - \frac{3}{16}$ and $\hat{\eta} = \eta - \frac{3}{8}$ vanish for conformally covariant operators.

and

$$\mathcal{S} = (-K - 2\omega_{;N})\, P_+ \,. \tag{B.24}$$

Tangential derivatives of the projection operators in this gauge are again zero

$$\chi_{|I} = 0\,, \qquad \chi_{|IJ} = 0\,, \qquad \chi_{|IJK} = 0\,, \tag{B.25}$$

as is the commutator of two covariant derivatives.

$$\Omega_{AB} = 0\,. \tag{B.26}$$

We can therefore consider the vector field as a collection of scalar fields, with the four tangential components having $X = \left(\omega'' - 2\omega'^2\right) e^{2\omega}$ and obeying Dirichlet boundary conditions. The normal component has $X = \left(2\omega'' - 2\omega'^2\right) e^{2\omega}$ and obeys Robin boundary conditions with $\mathcal{S} = -K - 2\omega_{;N}$.

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The ghost field operator is given by the variation of the gauge fixing term with respect to the gauge parameter. This gives us $X = \left(\omega'' - 2\omega'^2\right) e^{2\omega}$. BRST invariance requires the ghost fields to obey Dirichlet boundary conditions. The ghost fields therefore cancel the contribution to the effective action from two of the tangential components of the vector field.

The coefficients $c_i$ and $d_i$ for the gauge field are shown in table 1. Here, the effect of the ghost field has already been included.

### B.3 Fermion fields

The squared Dirac operators $\Delta = \mathcal{D}^*\mathcal{D}$ and $\Delta^* = \mathcal{D}\mathcal{D}^*$ are of Laplace type with

$$X = \frac{1}{4}R + m^2 \pm \partial\!\!\!/ m\,, \tag{B.27}$$

with the upper (lower) sign for $\Delta$ ($\Delta^*$). From now on, we consider the operator $\Delta$ only, as the results for $\Delta^*$ are identical.

We consider fermion mass terms of the form $m = b\omega' e^{\omega}$. The mass term for the fermion in the vector multiplet considered in section 4.3 is of this form with $b = 1/2$. The fermions obey boundary conditions $\Pi_-\psi = 0$, where

$$\Pi_\pm = \frac{1}{2}(1 \pm \Gamma^{11})\,. \tag{B.28}$$

To write this in a covariant form, we should write the projection operators $\Pi_\pm$ in terms of $\Gamma^N$. However, since the outward unit normal is of opposite sign on the two boundaries, we must define the projection operators

$$P_\pm = \begin{cases} \frac{1}{2}(1 \mp \Gamma^N) & \text{on } \partial\mathcal{M}^{(1)} \\ \frac{1}{2}(1 \pm \Gamma^N) & \text{on } \partial\mathcal{M}^{(2)} \end{cases} \tag{B.29}$$

The boundary condition $P_+\mathcal{D}\psi = 0$ can be written as $(\nabla_N - \mathcal{S})P_+\psi = 0$, where

$$\mathcal{S} = \left(-\frac{1}{2}K - m\right)P_+ \qquad \text{on } \partial\mathcal{M}^{(1)}\,, \qquad \mathcal{S} = \left(-\frac{1}{2}K + m\right)P_+ \qquad \text{on } \partial\mathcal{M}^{(2)}\,. \tag{B.30}$$

Tangential derivatives of the projection operators are non-zero, for example, on $\partial\mathcal{M}^{(2)}$,

$$\chi_{|I} = \frac{1}{4}\Gamma_I K\,, \qquad \chi_{|IJ} = -\frac{K^2}{16}\chi\eta_{IJ}\,, \qquad \chi_{|IJK} = -\frac{K^3}{64}\eta_{IJ}\Gamma_K\,. \tag{B.31}$$

The commutator of two covariant derivatives is also non-zero,

$$\Omega_{AB} = iR_{AB}{}^{CD}\Sigma_{CD} \qquad \Sigma_{CD} = -\frac{i}{8}[\Gamma_C, \Gamma_D]\,. \tag{B.32}$$

After inserting these into the general form for the heat kernel coefficient, we find the results for the $c_i$ and $d_i$ listed in table 1.



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